Journal of Aircraft

VOLUME 8 APRIL 1971 NUMBER 4

Predictions of Vortex-Lift Characteristics by a Leading-Edge Suction Analogy

EDWARD C. POLHAMUS*
NASA Langley Research Center, Hampton, Va.

A leading-edge suction analogy is used to develop analytical methods of predicting the low-speed lift and drag-due-to-lift characteristics of sharp-edge delta and delta related wing planforms. In addition, the method is extended to supersonic speeds and correlations made with experimental data. From the results, it appears that the leading-edge suction analogy accurately predicts the lift and drag-due-to-lift characteristics for conditions where essentially complete flow reattachment occurs inboard of the leading-edge vortices. For delta wings, the analogy indicates that the vortex lift is relatively independent of aspect ratio in the range of usual interest. The application of the analogy to nondelta wings has indicated that for the flow reattachment condition, arrow and double delta planforms produce greater values of vortex lift than the delta. Extension of the analogy to supersonic speeds provides a method which appears to accurately predict the reduction in vortex lift with increasing Mach number.

Nomenclature†

A = wing aspect ratio, b^2/S
a = longitudinal distance of root trailing edge ahead of tip
b = wing span
C_D = drag coefficient
C_{D0} = drag coefficient at zero lift
ΔC_D = drag-due-to-lift coefficient $(C_D - C_{D0})$
C_L = total lift coefficient, $C_{L_p} + C_{L_v}$
C_{L_p} = potential flow-lift coefficient
C_{L_v} = vortex-lift coefficient
C_{N_v} = vortex normal force coefficient
C_{S_p} = potential flow leading-edge suction coefficient
C_{T_p} = potential flow leading-edge thrust coefficient
E = elliptic integral of second kind where modulus = $[1 - (\beta \cot\Lambda_{LE})^2]^{1/2}$
K_p = constant in potential flow lift equation
K_v = constant in vortex-lift equation
M = Mach number
S = wing area
$T.E.$ = wing trailing edge
V_0 = velocity in flight direction
X_t = longitudinal distance from apex to tip
α = angle of attack
β = $(M^2 - 1)^{1/2}$
Λ_{LE} = wing leading-edge sweepback angle
Λ_i = Λ_{LE} of inboard wing panel
2-D = two dimensional

Presented as Paper 69-1133 at the AIAA 6th Annual Meeting and Technical Display, Anaheim, Calif., October 20–24, 1969; submitted November 17, 1969; revision received March 30, 1970. The author wishes to acknowledge L. Romanoski, formerly of the Subsonic Aerodynamics Branch, who carried out the computations for the nondelta wings.

* Head, Subsonic Aerodynamics Branch. Associate Fellow AIAA.

† All aerodynamic coefficients are nondimensionalized by dividing the forces by the product of the dynamic pressure and total wing planform area.

Introduction

THROUGHOUT the history of aeronautics, one of the major wing design considerations has been the avoidance of flow separation. However, as wing sweep angles were increased and the thickness decreased, to avoid undesirable compressibility effects, the maintenance of attached flow became increasingly difficult and the origin and spread of the separated flow was generally unpredictable causing many performance, stability, and control problems. Although many techniques have been developed to alleviate these problems, it has quite often been necessary to apply them by means of rather complicated variable geometry devices in order to satisfy the wide range of conflicting flow conditions encountered in the various regions of the flight envelope of modern high-speed aircraft. A rather historic departure from the time honored "attached flow" wing design concept occurred in the late 1950's when, based on studies made primarily at the Royal Aircraft Establishment, the British embarked on the design of a supersonic transport aircraft which was based on a slender, sharp-edge wing concept to minimize cruise wave drag but in which the flow (except for the cruise condition) was allowed to separate along the entire leading edge and produce the well-known leading-edge spiral vortex.[1,2] The primary advantages of this approach are that one type of stable flow can be maintained over a wide range of attitudes and Mach numbers without the need for flow control devices, and that the additional lift resulting from the leading-edge vortex flow tends to eliminate the need for high-lift devices. Competing with these advantages, of course, is the disadvantage of increased drag resulting from the loss of leading-edge thrust. The resulting British-French "Concorde" project, the similar Russian TU-144 design, and the interest in slender hypersonic vehicles have added impetus to the study of leading-edge vortex flows.

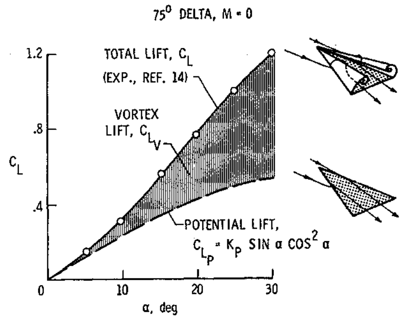

Fig. 1 Illustration of the vortex lift for a 75° delta wing.

The general nature of the vortex flow has been understood for many years, and several analytical approaches to the prediction of the aerodynamic characteristics for slender delta wings have been developed. These analytical methods, however, have primarily been based either on subsonic lifting-surface theory with crude approximations of the leading-edge vortex or on slender wing theory which is inappropriate at subsonic speeds because the trailing-edge Kutta condition is not satisfied, and the theory does not account for the effect of Mach number at supersonic speeds. A need, therefore, still exists for improved methods and extensions to nondelta type wings. The purpose of this paper, therefore, is to present an evaluation of a relatively new concept based on an analogy between the vortex induced lift on delta wings and the potential flow leading-edge suction which offers promise in avoiding the previous problems.[3,4] The resulting analytical method is extended to nondelta planforms and supersonic Mach numbers, and comparisons with experimental results are made. The present application deals only with wings having no camber and twist.

Description of Method

The methods to be developed in this section apply to wings having no camber and twist. In addition, it is assumed that the wings are thin and the leading edges of sufficient sharpness that separation is fixed at the leading edge and no leading-edge suction is developed. With regard to practical applications, it must be kept in mind that at full-scale Reynolds numbers even a small leading-edge radius could result in the development of some suction and a change in the separation point and vortex strength.

Figure 1 illustrates the general nature of the vortex flow and the magnitude of the lift increase associated with the flow induced by the vortex for a thin sharp-edge delta wing having leading-edges swept 75° ($A = 1.07$). As illustrated by the upper sketch on Fig. 1, the flow separates at the sharp leading edge and rolls up into two spiral vortex sheets. Air is drawn around these vortices and a reattached flow results, somewhat analogous to the reattachment behind a two-dimensional laminar separation bubble. The well-known nonlinear character of the vortex induced lift is apparent in the experimental results, and the large increase in lift over the potential flow values is illustrated by the shaded area. Since the vortex flow induces reattachment and a Kutta condition still exists at the trailing edge, it is assumed that the total lift coefficient consists of a potential flow term C_{L_p} and a vortex-lift term C_{L_v}. Although this assumption regarding the potential flow term is not new, the definition used in the present method is somewhat different from that usually used. The expression for C_{L_p} is derived in Ref. 3 and takes the following form:

$$C_{L_p} = K_p \sin\alpha \cos^2\alpha \quad (1)$$

where K_p is the normal force slope given by small disturbance potential flow lifting-surface theory, and the trigonometric terms account for the true boundary condition and the Kutta type condition at the leading edge (due to the leading-edge separation) which eliminates the leading-edge suction, thereby causing the normal force to become the resultant force. The use of lifting-surface theory rather than slender-wing theory appears mandatory due to the Kutta condition at the trailing edge.

The vortex-lift term is determined by the suction analogy described in Ref. 3 and illustrated in Fig. 2. In this approach, details of the real flow such as the primary and secondary vortex size, shape, and position, which have been difficult to represent sufficiently accurately by mathematical models, are ignored and only the flow reattachment feature is considered. Briefly stated, the analogy assumes that with reattached flow the normal force on the upper surface required to maintain the flow about the vortex is the same as the leading-edge suction force required to maintain the attached flow about the leading edge in the potential flow case. From this intuitive assumption, it follows that the normal force increment associated with the leading-edge vortex flow can be predicted from potential flow leading-edge suction calculations. It can then be shown that

$$C_{L_v} = K_v \sin^2\alpha \cos\alpha \quad (2)$$

where

$$K_v = \partial C_{S_p}/\partial\alpha^2 = (\partial C_{T_p}/\partial\alpha^2)(1/\cos\Lambda_{LE}) \quad (3)$$

The relationship between the leading-edge thrust C_{T_p} and the leading-edge suction C_{S_p} is illustrated in Fig. 3. For wings with unbroken leading edges, it is shown in Ref. 3 that the thrust and, therefore, K_v can be determined simply from the overall lift and induced drag obtained from an accurate lifting-surface theory such as Ref. 5 (for subsonic speeds). For broken leading-edge wings such as double deltas, however, the over-all induced drag cannot be used and it is necessary to determine the thrust on the inner and outer panels separately or calculate the over-all suction directly. For this case, the method of Ref. 6 has been found to be most appropriate for determining K_v at subsonic speeds. Methods for predicting K_p and K_v at supersonic speeds will be discussed in a later section.

The total lift is given by the sum of Eqs. (1) and (2).

$$C_L = K_p \sin\alpha \cos^2\alpha + K_v \sin^2\alpha \cos\alpha \quad (4)$$

Throughout the figures the term "present theory" refers to Eq. (4) with the appropriate lifting-surface values of K_p and K_v.

With regard to drag-due-to-lift, it can be shown that with fully developed leading-edge vortex flow (complete loss of leading-edge thrust) on a wing with no camber or twist

$$\Delta C_D/C_L^2 = (C_L \tan\alpha/C_L^2) = \tan\alpha/C_L \quad (5)$$

where C_L is determined from Eq. (4).

Inasmuch as the present method does not provide information with regard to the exact manner in which the potential

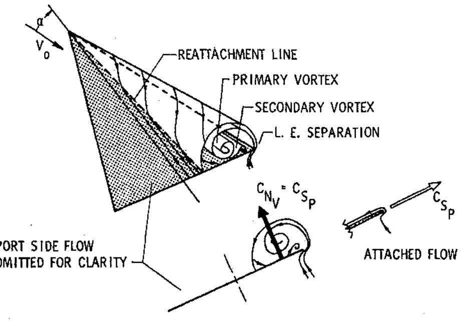

Fig. 2 Illustration of vortex flow details and leading-edge suction analogy.

flow part of the total lift is redistributed over the wing surface, no detailed analysis of load distribution or pitching moments will be made. There are, however, two observations of interest. First, it can be shown that for slender delta wings the longitudinal center of the leading-edge suction is very near the potential flow aerodynamic center and secondly, that the experimental aerodynamic center for wings with vortex flow is not far from the potential flow position. From this it would appear that the redistribution of the potential flow part of the total lift is primarily spanwise. Reference 7 presents some interesting results in this connection with regard to slender delta wings with longitudinal camber.

Delta Wings in Incompressible Flow

The potential flow lift constant K_p and the vortex lift constant K_v for delta wings in incompressible flow (obtained from Ref. 3) are shown in Fig. 4 as a function of aspect ratio. Also shown are the corresponding leading-edge sweep values. It is interesting to note that while K_p shows the expected rapid increase with aspect ratio K_v increases only slightly with aspect ratio. The relatively constant value of K_v is associated with the fact that the increase in the leading-edge thrust parameter $\partial C_{T_p}/\partial \alpha^2$ which accompanies an increase in aspect ratio is offset by the decrease in $1/\cos\Lambda_{LE}$ [see Eq. (3)].

Figure 5 presents a comparison between measured lift (Refs. 8–10) and the lift predicted by Eq. (4) using the values of K_p and K_v from Fig. 4 for a series of sharp-edge delta wings. To illustrate the magnitude of the vortex lift, the potential lift C_{L_p} from Eq. (1) is also presented. The results indicate that the vortex lift is very well predicted by the present method up to angles of attack of 20° or greater. Comparisons over a wider range of aspect ratios are presented in Ref. 3.

A comparison of the drag-due-to-lift parameter for the same series of wings is presented in Fig. 6. The solid lines represent the present theory as determined from Eq. (5) with C_L from Eq. (4). Also shown is the potential flow value for an elliptic span loading $1/\pi A$ and a line representing the assumption of zero leading-edge thrust but with no vortex lift included [Eq. (5) with C_L from Eq. (1)]. There are several items of interest with regard to the drag-due-to-lift characteristics of these slender sharp-edge wings. First, it will be noted that the experimental values are in good agreement with the present theory. One of the most prominent effects is the reduction in the drag-due-to-lift factor with increasing lift coefficient. This is associated, of course, with the $\sin^2\alpha$ factor in the vortex-lift term and becomes more pronounced as the aspect ratio is reduced because of the fact that the vortex lift becomes a larger part of the total lift. Another item of interest is that the penalty because of the loss of leading-edge thrust is not nearly so severe as would be predicted by zero thrust theory with the vortex lift omitted. This fact has implications with regard to the magnitude of benefits to be derived from the use of camber and twist to recover the leading-edge thrust.[4,11]

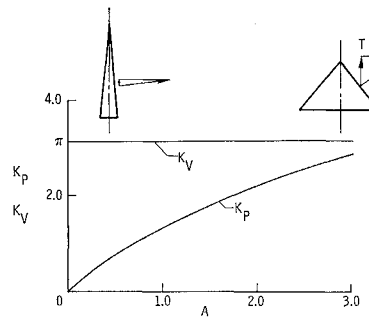

Fig. 4 Variation of K_p and K_v with A for delta wings; $M = 0$.

Effect of Flow Breakdown

Since the present method is based on an analogy with potential flow leading-edge suction, which requires flow reattachment inboard of the vortex, it would be expected to diminish in accuracy as various types of flow breakdown begin to limit the reattachment. All wings will eventually reach an angle of attack where flow reattachment fails to occur and a Helmholtz type of flow will exist. However, within the normal operating angle-of-attack range of aircraft, the pronounced vortex flow breakdown effects on delta wings are limited to the "not-so-slender" (or high aspect ratio) and the "extremely slender" (or low aspect ratio) wings.

The type of vortex flow breakdown which is most pronounced on the high aspect ratio wings is that of vortex breakdown at the wing trailing edge. The leading-edge vortex core experiences an abrupt expansion at some point in the wake downstream of the wing trailing edge, and as the wing angle of attack increases, this so-called "breakdown" point moves forward and eventually reaches the trailing edge. There have been numerous studies of these phenomena,[12–14] and it has generally been concluded that important effects on the wing characteristics do not occur until the "breakdown" reaches the trailing edge and that the angle of attack at which this occurs decreases with increasing aspect ratio. An indication of the effect of vortex breakdown in relation to the leading-edge suction analogy is presented in Fig. 7 for both the lift and drag-due-to-lift of an aspect ratio 2.0 delta wing. The experimental data were obtained from Ref. 8. The vortex breakdown point[14] is indicated by the arrowhead symbol and it can be seen that no serious departures from the present theory occur until the angle of attack for vortex breakdown is reached. The departures are presumed to be associated with the failure of the flow to reattach in the region of the vortex breakdown. Another possible phenomenon contributing to incomplete recovery of leading-edge suction on the upper surface is the helical nature of the flow about the vortex which would appear to require wing area aft of the tip to provide complete recovery (see sketch on Fig. 7). The loss of lift due

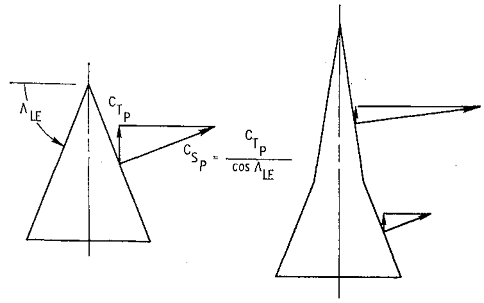

Fig. 3 Effect of leading-edge sweep on relationship between thrust and suction force.

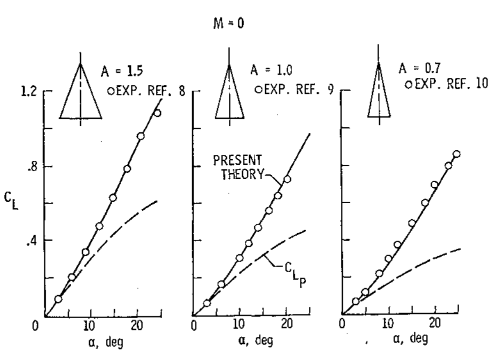

Fig. 5 Comparison of present theory with experimental lift of delta wings; $M = 0$.

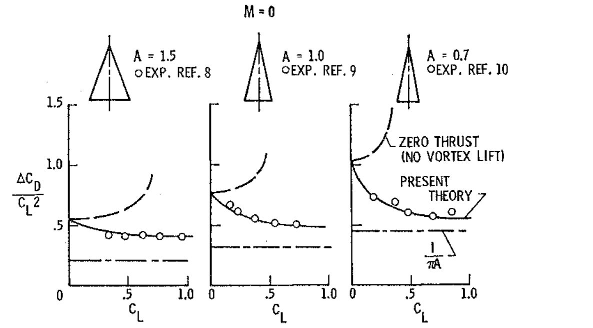

Fig. 6 Comparison of present theory with experimental drag-due-to-lift of delta wings; $M = 0$.

to this incomplete recovery would be expected to be greatest for the higher aspect ratios.

An extremely slender wing phenomenon such as vortex contact[15] or vortex asymmetry[16] can occur before vortex breakdown. Both of these effects can eliminate the flow reattachment, thereby reducing the lift. Figure 8 illustrates the effect of vortex asymmetry on an aspect ratio $\frac{1}{4}$ delta wing ($\Lambda_{LE} = 86.5°$) based on an experiment by J. E. Lamar and W. P. Phillips of the Langley Research Center. The results indicate that the lift falls below the suction analogy above about 15° angle of attack which corresponded to the occurrence of a large rolling moment. It is of interest that for this extremely slender wing value of drag-due-to-lift considerably less than $1/\pi A$ are obtained even though the leading-edge thrust has been lost. This effect is discussed in Ref. 4.

A summary of the effects of the various types of flow breakdown is presented in Fig. 9 in the form of two angle-of-attack-aspect-ratio boundaries below which the recovery of leading-edge suction as vortex normal force is essentially complete. Under this condition the suction analogy provides a very accurate estimate of the vortex lift. The vortex breakdown boundary is a well established one obtained from Ref. 14, while the vortex contact or asymmetry boundary is a preliminary one based on rolling-moment data, tuft grid pictures, and data analysis. Also shown for reference is the angle of attack for which two-dimensional bubble bursting occurs on thin sharp-edge sections.[17]

Nondelta Wings in Incompressible Flow

The leading-edge suction analogy can, of course, be applied to any fully tapered planform and it is the purpose of this section to evaluate its accuracy for two series of nondelta planforms. Since some of the wings considered require a knowledge of the spanwise distribution of thrust, Ref. 6 was used to calculate K_p and K_v in this section.

Notch-Ratio Series

Figure 10 presents the calculated values of K_p and K_v as a function of trailing-edge notch ratio for a series of fully tapered

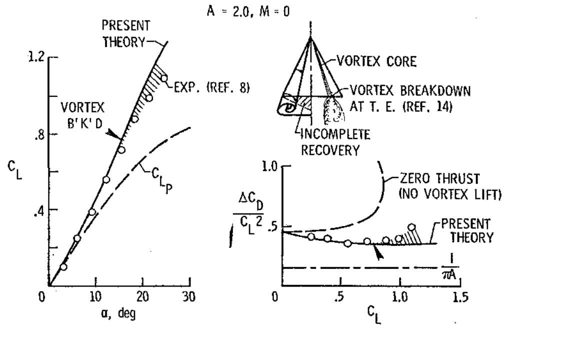

Fig. 7 Effect of vortex breakdown and incomplete suction recovery on characteristics of $A = 2$ delta wing; $M = 0$.

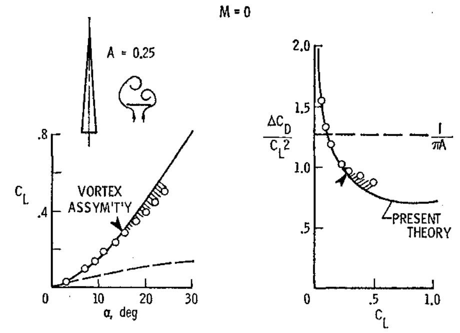

Fig. 8 Effect of vortex asymmetry on characteristics of $A = \frac{1}{4}$ delta wing; $M = 0$.

wings having leading-edge sweep angles of 70°. The results indicate a large effect of notch ratio (or aspect ratio) on the vortex-lift constant K_v with an increase in K_v occurring as the planform progresses from the diamond planform to the arrow planform. It is of interest to note that this method of increasing aspect ratio results in K_v variations which are considerably greater than for the delta wing aspect-ratio series presented in Fig. 4. This is due primarily to the fact that by notching the wing to increase aspect ratio the ratio of leading-edge suction to leading-edge thrust remains constant, while increasing aspect ratio by reducing the leading-edge sweep of a delta wing results in a reduction in the suction-to-thrust ratio.

An evaluation of the accuracy of the leading-edge suction analogy method in predicting the vortex lift for wings of various notch ratios is presented in Fig. 11 for wings having leading-edge sweep angles of 70°. The experimental results were obtained from Ref. 14. From the comparison, it appears that the large variation in vortex-lift constant with notch ratio predicted by the leading-edge suction analogy is reasonably well substantiated by the experimental results up to an angle of attack of about 12°. At higher angles of attack, the diamond planform develops greater lift while the arrow develops less lift than that predicted by the suction analogy. For the diamond wing the increase above the present theory at high angles of attack is probably associated with the increase in wing area downstream of the wing tip that is affected by the vortex as the vortex size increases. The reduction in measured lift at high angles of attack relative to the theory for the arrow wing is due to the fact that as the vortex size increases there is insufficient wing chord to allow flow reattachment and full lift development. In addition to the lift loss, this effect results in the well-known pitch-up problem of arrow wings.

Double-Delta Series

To illustrate the effect of broken leading edges, Fig. 12 presents the calculated values of K_p and K_v for a series of double-delta wings. The outer panel sweep angle was held constant at 65° and the inner panel sweep was varied with the break location remaining fixed at 31% of the wing semi-

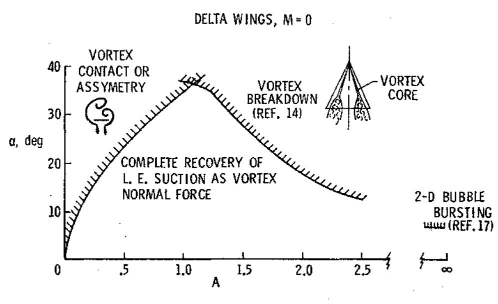

Fig. 9 Leading-edge suction recovery boundaries for delta wings; $M = 0$.

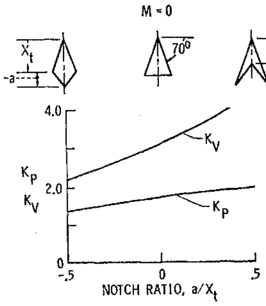

Fig. 10 Variation of K_p and K_v with notch ratio; Λ_{LE} = 70°, M = 0.

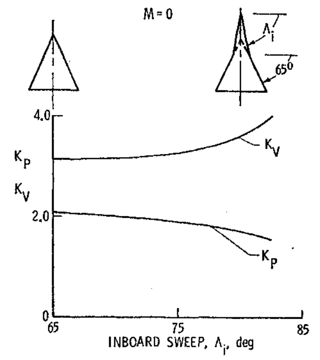

Fig. 12 Variation of K_p and K_v with inboard sweep angle for double delta wings (break at 31% semispan); M = 0.

span. The values of K_p and K_v are presented as a function of the inner panel sweep angle with the value of 65° representing the pure delta wing. As would be expected, the value of the potential flow lift constant K_p decreases with increasing inner panel sweep due to the combined effects of increased sweep and reduced aspect ratio. It is interesting to note, however, that the value of the vortex lift constant K_v increases with increasing inner panel sweep angle with the increase being especially rapid for inner panel sweep angles above 75°.

This increase, which occurs despite a slight reduction in leading-edge thrust (not shown), is associated with the sizable increase in leading-edge length and the corresponding increase in the resultant suction force as illustrated in Fig. 3.

Figure 13 presents a correlation of experimental data (obtained from Ref. 14) with the present theory for two double-delta wings and a delta wing all having 65° of leading-edge sweep in the outer panel. The pure delta wing and the double-delta having the 80° inner panel sweep are a part of the systematic series of Fig. 13 while the double-delta wing having the 75° inner sweep differs in that its break location corresponds to 47% semispan station. The theoretical values of the vortex-lift constant K_v are listed by each planform sketch. The agreement between the experimental measurements and the present theory is rather good, and in fact, at the higher angles of attack, it is superior to that obtained with the pure 65° delta wing. This improvement for the double deltas is presumed to be associated with the fact that the relative portion of the vortex lift carried on the inboard part of the wing is greater for the double deltas, thereby resulting in a more complete recovery of suction as vortex normal force. Although the variation of the vortex-lift constant is relatively small for this series of double-delta wings, the agreement between experiment and the present method appears good enough to assume, at least tentatively, that the leading-edge suction analogy will be useful in predicting the vortex lift of double-delta wings.

Delta Wings at Supersonic Speeds

The main features of the subsonic flow over slender sharp-edge delta wings are maintained well into the supersonic speed range. However, in contrast to the subsonic case where the vortex lift is relatively independent of aspect ratio and Mach number, it has been found that the supersonic vortex lift is critically dependent on both aspect ratio (leading-edge sweep angle) and Mach number. For example, experimental data in Ref. 18 indicate that both the magnitude of the vortex lift and the size of the leading-edge vortex decreases as the supersonic Mach number is increased. It is because of this strong dependence on both aspect ratio and Mach number that the existing supersonic analytical approaches based on slender-body theory are seriously limited in application and empirical approaches[19] or methods based on combining slender body and linear wing theory approximations[20] have been resorted to for the over-all lift. Although slender-body methods are inadequate for predicting the over-all forces, they have provided reasonable predictions of the general distribution of load with the method of Ref. 21 probably being the most notable contribution.

In order to determine the applicability of the leading-edge suction analogy to the supersonic case, existing potential flow theories have been used to predict the effect of Mach number and aspect ratio on the constants K_p and K_v. Based on the exact linearized theory of Stewart (see Ref. 22 or 23), it can be shown that

$$K_p = \pi A/2E \tag{6}$$

If the solution for the leading-edge thrust given by Brown[24] and the expression for cos Λ_{LE} in terms of A is substituted in Eq. (3), the following expression for the vortex-lift constant can be obtained:

$$K_v = \pi[(16 - (A\beta)^2)(A^2 + 16)]^{1/2}/16E^2 \tag{7}$$

The variation of K_p and K_v with supersonic Mach number as predicted by Eqs. (6) and (7) for a delta wing having an aspect ratio of 1.0 (Λ = 76°) is presented in Fig. 14. The subsonic variations were obtained by application of the Prandtl-Glauert transformation to the subsonic suction calculation method. Of particular interest is the variation of K_v which is characterized by a slight increase up to M = 1 followed by a rather sharp decrease with a value of zero being

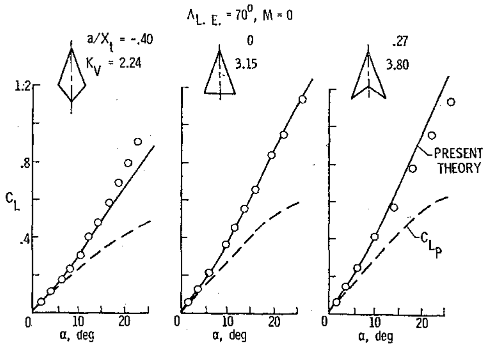

Fig. 11 Comparison of present theory with experimental lift of wings of various notch ratios (data from Ref. 14); M = 0, Λ_{LE} = 70°.

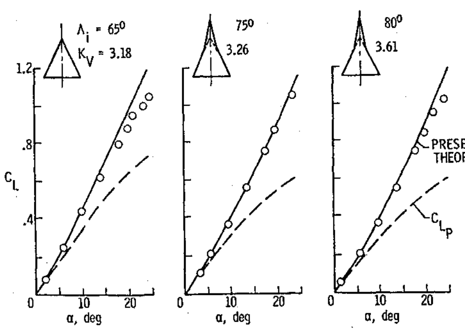

Fig. 13 Comparison of present theory with experimental lift of double-delta wings (data from Ref. 14); Λ_{LE} = 65°, M = 0.

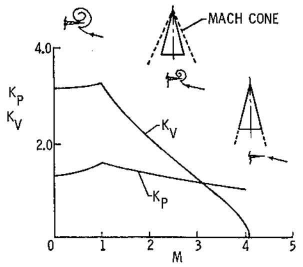

Fig. 14 Variation of K_p and K_v with Mach number for A = 1 delta wing.

reached at the Mach number for which the Mach cone coincides with the wing leading edge. This variation is a reflection of the decrease in potential flow leading-edge thrust as the upwash ahead of the leading-edge decreases and the lower surface stagnation point moves forward until it reaches the leading edge at the sonic leading-edge Mach number. The manner in which the leading-edge suction analogy relates the potential flow suction to the vortex-lift constant K_v is illustrated by the sketches in Fig. 14. The forward shift of the lower surface stagnation point with increasing supersonic Mach number results in a smaller, weaker vortex which finally vanishes when, at the Mach number corresponding to a sonic leading edge, the stagnation point lies along the leading edge and no flow reversal occurs around the edge. For higher aspect ratios the reduction in K_v with Mach number is more rapid while for lower aspect ratios it is more gradual.

An evaluation of the leading-edge suction analogy is presented in Fig. 15 where the resulting variation of lift coefficient with angle of attack is compared with experimental data for a thin sharp-edge delta wing of aspect ratio 1.0. The incompressible results from Fig. 5 are repeated for comparison and the supersonic experimental results were obtained from Refs. 25 and 26. The magnitude of the leading-edge vortex lift is represented by the difference between the total lift (solid line for theory and symbols for experiment) and C_{L_p} which is obtained from Eq. (1) with K_p based on Stewart's exact linearized potential flow theory for the normal force slope [Eq. (6)]. The experimental results substantiate, in general, the reduction in vortex lift with increasing Mach number predicted by the suction analogy and are in reasonably good quantitative agreement. An additional point of interest is the fact that for both supersonic Mach numbers the experimental lift slope is reasonably linear and at the highest Mach number the theory is also reasonably linear. This linearity when observed alone could be interpreted as an indication of the absence of vortex lift. However, when compared with the lift term C_{L_p} which represents the case of zero thrust but no vortex lift, it is clear that the vortex lift is present and that its nonlinear characteristics is offset by the opposite nonlinear trend of the C_{L_p} term associated with the trigonometric terms in Eq. (1). It should be mentioned that at high Mach numbers and angles of attack the upper surface vacuum limit will affect the lift. However, a knowledge of the surface loading is required before the limit can be predicted.

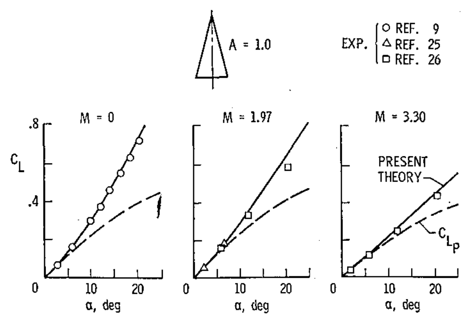

Fig. 15 Comparison of present theory with experimental lift for various Mach numbers; A = 1 delta wing.

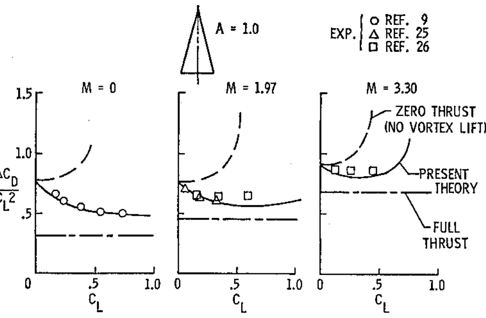

Fig. 16 Comparison of present theory with experimental drag-due-to-lift for various Mach numbers; A = 1 delta wing.

The effect of the vortex flow on the supersonic drag-due-to-lift is illustrated in Fig. 16 where the parameter $\Delta C_D/C_L^2$ is presented as a function of lift coefficient. In addition to the experimental values (obtained from the experimental C_L $\tan\alpha$), the results obtained from three analytical approaches are shown for comparison. That labeled "full thrust" represents the linearized potential flow solution including the leading-edge thrust by the method of Ref. 24 and is presented only to indicate the probable drag penalty associated with the leading-edge separation vortex. The other two curves are based on the fact that no leading-edge thrust would be expected on thin sharp-edge wings. That labeled "zero thrust (no vortex lift)" is obtained from C_{L_p} $\tan\alpha$ with K_p being obtained from Stewart's exact linearized theory and represents the often used but unrealistic assumption of no leading-edge thrust and no vortex lift. The curve labeled "present theory" includes the effect of the vortex lift in reducing the angle of attack for a given lift coefficient. The supersonic results are quite similar to the subsonic in that because of the vortex lift the drag-due-to-lift parameter decreases with increasing lift coefficient and that the values are considerably less than predicted from linearized theory with zero thrust. The results also indicate, as in the subsonic case, that the incremental drag reductions available through camber and twist are considerably less than predicted by methods which ignore the vortex-lift effect on the zero suction case.

Additional Applications

Although this paper has dealt only with wings having no camber or twist, the importance of the leading vortex flow at "off design" conditions on cambered and twisted wings[27,28] should not be overlooked and a reliable method of accounting for these effects is needed. In this regard, the possibility of utilizing the suction analogy to aid in developing an analytical method for cambered and twisted wings should be explored. The analogy also offers the possibility of predicting the rolling moment due to sideslip and the roll and pitch damping derivatives of slender sharp-edge wings.

Concluding Remarks

The analogy between the potential flow leading-edge suction and the leading-edge vortex lift on slender sharp-edge wings has been applied to the prediction of the low-speed vortex lift characteristics of delta and nondelta planforms and the results compared with experimental data. In addition, the resulting analytical method has been extended to supersonic speeds for delta wings.

From the results, it appears that the leading-edge suction analogy accurately predicts the lift and drag-due-to-lift characteristics of slender, sharp-edge fully tapered wings for conditions where essentially complete flow reattachment occurs inboard of the leading-edge vortices. For delta wings, the analogy indicates that the vortex lift is relatively independent of aspect ratio in the range of usual interest. The application of the analogy to nondelta wings has indicated that, for the flow reattachment condition, arrow and double-delta planforms produce greater values of vortex lift than the delta. Extension of the analogy to supersonic speeds provides a method which appears to accurately predict the reduction in vortex lift with increasing Mach number.

Although the analogy does not predict the surface load distribution, it is believed that the possibility of using the analogy to establish an improved mathematical model for predicting load distributions is worthy of investigation. Finally, it appears that a study of the analogy in connection with the vortex flow encountered in the "off-design" conditions for cambered and twisted wings should be pursued as well as application to static and dynamic-stability derivatives.

References

[1] Maltby, R. L., "The Development of the Slender Delta Concept," *Aircraft Engineering*, March 1968.

[2] Küchemann, D. and Weber, J., "An Analysis of Some Performance Aspects of Various Types of Aircraft Designed to Fly Over Different Ranges at Different Speeds," *Progress in Aeronautical Sciences 9*, Pergamon Press, New York, 1968, pp. 329–456.

[3] Polhamus, E. C., "A Concept of the Vortex Lift of Sharp-Edge Delta Wings Based on a Leading-Edge Suction Analogy," TN D-3767, Dec. 1966, NASA.

[4] Polhamus, E. C., "Application of the Leading-Edge Suction Analogy of Vortex Lift to the Drag-Due-to-Lift of Sharp-Edge Delta Wings," TN D-4739, Aug. 1968, NASA.

[5] Lamar, J. E., "A Modified Multhopp Approach for Predicting Lifting Pressures and Camber Shape for Composite Planforms in Subsonic Flow," TN D-4427, July 1968, NASA.

[6] Wagner, S., "On the Singularity Method of Subsonic Lifting-Surface Theory," AIAA Paper 69-37, New York, 1969.

[7] Nangia, R. K. and Hancock, G. J., "Delta Wings With Longitudinal Camber at Low Speed," ARC 31-588, Sept. 1969, Aeronautical Research Council.

[8] Barlett, G. E., and Vidal, R. J., "Experimental Investigation of Influence of Edge Shape on the Aerodynamic Characteristics of Low Aspect Ratio Wings at Low Speeds," *Journal of Aeronautical Sciences*, Vol. 22, No. 8, Aug. 1955, pp. 517–533, 588.

[9] Peckham, D. H., "Low-Speed Wind-Tunnel Tests on a Series of Uncambered Slender Pointed Wings With Sharp Edges," RM 3186, 1961, British Aeronautical Research Council, England.

[10] Fink, P. T., "Wind-Tunnel Tests on a Slender Delta Wing at High Incidence," *Zeitschrift für Flugwissenschaften*, Jahrg. 4, Heft 7, July 1956, pp. 247–249.

[11] Kuchemann, D., "Note on the Drag-Due-to-Lift of Slender Wings," *Journal of Royal Aeronautical Society*, Vol. 61, No. 558, June 1957, pp. 423–424.

[12] Poisson-Quinton, P. and Erlich, E., Hyperlift and Balancing of Slender Wings," TT F-9523, Aug. 1965, NASA.

[13] Earnshaw, P. B., "Measurements of Vortex-Breakdown Position at Low Speed on a Series of Sharp-Edged Symmetrical Models," RAE TR 64047, Nov. 1964, Royal Aeronautical Establishment.

[14] Wentz, W. H., Jr., "Wind-Tunnel Investigations of Vortex Breakdown on Slender Sharp-Edged Wings," CR-98737, 1968, NASA.

[15] Maltby, R. L. and Peckham, D. H., "Low-Speed Flow Studies of the Vortex Patterns Above Inclined Slender Bodies Using a Smoke Technique," RAE Addendum to TN Aero 2482, March 1957, Royal Aeronautical Establishment.

[16] Bird, J. D., "Tuft-Grid Surveys at Low Speeds for Delta Wings," TN D-5045, Feb. 1969, NASA.

[17] McCullough, G. B., and Gault, D. E., "Examples of Three Representative Types of Airfoil-Section Stall at Low Speed," TN 2502, Sept. 1951, NACA.

[18] Squire, L. C., Jones, J. G., and Stanbrook, A., "An Experimental Investigation of the Characteristics of Some Plane and Cambered 65° Delta Wings at Mach Numbers From 0.7 to 2.0," ARE RM 3305, 1963, Aeronautical Research Council.

[19] Collingbourne, J. R., "An Empirical Prediction Method for Non-Linear Normal Force on Thin Wings at Supersonic Speeds," ARE Current Papers No. 662, 1963, Aeronautical Research Council.

[20] Squire, L. C., "The Estimation of the Non-Linear Lift of Delta Wings at Supersonic Speeds," *Journal of the Royal Aeronautical Society*, Vol. 67, Aug. 1963, pp. 476–480.

[21] Smith, J. H. B., "Improved Calculations of Leading-Edge Separation From Slender Delta Wings," TR 66070, March 1966, Royal Aircraft Establishment.

[22] Stewart, H. J., "The Lift of a Delta Wing at Supersonic Speeds," *Quarterly of Applied Mathematics*, Vol. IV, No. 3, Oct. 1946, pp. 246–254.

[23] Jones, R. J., "Estimated Lift-Drag Ratios at Supersonic Speed," TN 1350, July 1947, NACA.

[24] Brown, C. E., "Theoretical Lift and Drag of Thin Triangular Wings at Supersonic Speeds," Rept. No. 839, Nov. 1946, NACA.

[25] Stahl, W., "Zum nichtlinearen Verhalten des Auftriebes schlanker, ebener Deltaflügel in Ueberschallströmung," Bericht 67 A 44, 1968.

[26] Hill, W. A., Jr., "Experimental Lift of Low-Aspect-Ratio Triangular Wings at Large Angles of Attack and Supersonic Speeds," RM A57I17, Nov. 1957.

[27] Carlson, H. W., "Pressure Distributions at Mach Numbers 2.05 on a Series of Highly Swept Arrow Wings Employing Various Degrees of Twist and Camber," TN D-1264, 1962, NASA.

[28] Smith, J. H. B. et al., "The Calculation of the Warp to Produce a Given Load and the Pressures Due to a Given Thickness on Thin Slender Wings in Supersonic Flow," RM 3471, 1967, Ministry of Aviation, Aeronautical Research Council.